I. Conventional three-neutrino oscillations

II. Models with extra sterile neutrinos

Conclusions
I. Conventional three-neutrino oscillations

General three-neutrino framework

- Equation of motion: 6 parameters (including CP-violating effects):

\[
i \frac{d\vec{\nu}}{dt} = H \vec{\nu}; \quad H = O \cdot H_0^d \cdot O^\dagger + V;
\]

\[
O = \begin{pmatrix}
    c_{12} & c_{13} & s_{12} c_{13} \\
    s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta_{cp}} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta_{cp}} & s_{13} e^{-i\delta_{cp}} \\
    s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta_{cp}} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i\delta_{cp}} & c_{13} c_{23}
\end{pmatrix}, \quad \vec{\nu} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix};
\]

\[
H_0^d = \frac{1}{2E_\nu} \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2); \quad V = \text{diag}(\pm \sqrt{2} G_F N_e, 0, 0).
\]

The hierarchy approximation

- From SOL and ATM data, we have $\Delta m_{21}^2 \ll \Delta m_{31}^2$;
- SOL: $\Delta m_{31}^2 \approx \infty \Rightarrow$ only 3 parameters: $\Delta m_{21}^2, \theta_{12}, \theta_{13}$;
- ATM: $\Delta m_{21}^2 \approx 0 \Rightarrow$ only 3 parameters: $\Delta m_{31}^2, \theta_{23}, \theta_{13}$;
- CP-violating phase $\delta_{cp}$ disappears from equations;
- Chooz limit: $\theta_{13} \approx 0 \Rightarrow$ SOL and ATM decoupled.
I. Conventional three-neutrino oscillations

Solar sector: the $\theta_{13} = 0$ limit

- Most relevant changes since last year:
  - new SNO-lreta (low energy threshold analysis) replaces separated SNO-I and SNO-II data sets ⇒ bounds on $\theta_{12}$ enhanced;
  - inclusion of Borexino results (both $^7$Be and $^8$B data) ⇒ $\theta_{12}$ decreases;
  - new results from SAGE with reduced systematics ⇒ negligible effects;
  - new AGSS09 solar model ⇒ $\theta_{12}$ increases;
- $\Delta m^2_{21}$ determined by KamLAND ⇒ insensitive to details of solar data.
I. Conventional three-neutrino oscillations

Bound on \( \theta_{13} \) from solar & KamLAND

- \( \nu_\mu \equiv \nu_\tau \Rightarrow \) no sensitivity to \( \theta_{23} \) and \( \delta_{\text{CP}} \);
- \( \Delta m_{31}^2 \approx \infty \Rightarrow \) specific \( \Delta m_{31}^2 \) value irrelevant;
- data only depend on \( \Delta m_{21}^2, \theta_{12} \) and \( \theta_{13} \);
- \( P_{ee} \approx \begin{cases} \text{Kam: } \cos^4 \theta_{13} \left( 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 L}{4E} \right), \\
\text{low-E: } \cos^4 \theta_{13} \left( 1 - \frac{1}{2} \sin^2 2\theta_{12} \right), \\
\text{high-E: } \cos^4 \theta_{13} \sin^2 \theta_{12}; \end{cases} \)
- as \( \theta_{13} \) increases, \( \cos^4 \theta_{13} \) decreases and:
  - KamLAND and low-E data favor smaller \( \theta_{12} \);
  - high-E data favor larger \( \theta_{12} \) and \( \Delta m_{21}^2 \);
  - Kam fit gets worse as osc. are suppressed;
- synergy between solar and KamLAND data: as \( \theta_{13} \) increases, \( \Delta m_{21}^2 \) increases in solar data; remains stable in KamLAND; hence, a tension appear.
I. Conventional three-neutrino oscillations

Hint for non-zero $\theta_{13}$ in solar and KamLAND data

- For $\theta_{13} = 0$, we have $\sin^2 \theta_{12} = \begin{cases} 0.30 \text{ from solar data;} \\ 0.36 \text{ from KamLAND data;} \end{cases}$

- hence there is a tension between solar and KamLAND;

- as we have just seen, when $\theta_{13}$ increases:
  - solar region slightly moves to larger $\theta_{12}$ (high-E data dominate over low-E ones);
  - KamLAND region definitely shifts to smaller $\theta_{12}$;

- therefore, a non-zero value of $\theta_{13}$ reduces the tension between solar and KamLAND data;

- new SNO-III data favor smaller $\phi_{CC}/\phi_{NC} \Rightarrow$ smaller $\theta_{12}$ from solar $\Rightarrow$ tension with KamLAND is increased $\Rightarrow$ larger $\theta_{13}$ favored.

\begin{align*}
\Delta m^2_{21} &= 7.58 \times 10^{-5} \text{ eV}^2 \\
\Delta \chi^2_{\text{KamLAND}} \quad \text{Solar} \quad \text{Solar + KamLAND}
\end{align*}
I. Conventional three-neutrino oscillations

Atmospheric sector: the $\Delta m_{21}^2 = 0$ limit

- $\Delta m_{31}^2$ is now determined by Minos-DIS ($\nu_\mu$) data;
- $\theta_{23}$ still dominated by atmospheric data;
- $\theta_{13}$ mainly bounded by Chooz, with small atm contribution;
- hint of non-zero $\theta_{13}$ from preliminary Minos-APP ($\nu_e$) data;
- Minos antineutrino ($\bar{\nu}_\mu$) data agree with $\nu$ data at 90% CL.
I. Conventional three-neutrino oscillations

**Hint of non-zero $\theta_{13}$ in atmospheric data?**

- Possible hint of non-zero $\theta_{13}$ in atm SK-I (+ LBL) data; [Fogli et al, hep-ph/0506083 & arXiv:0806.2649];
- hint linked to a signature of SK-I data not present in SK-II ⇒ check SK-III;
- its appearance strongly depends on the details of the simulation ⇒ only SK collab. can say a final word!
I. Conventional three-neutrino oscillations

Subleading effects in the ATM sector

- Present reactor and accelerator data dominate $|\Delta m_{31}^2|$ and $\theta_{13}$ but give no info on:
  - the mass hierarchy (sign of $\Delta m_{31}^2$);
  - the octant (sign of $\theta_{23} - \pi/4$);
  - the CP phase;
- note the high degree of symmetry of the gray regions;
- conversely, regions including ATM are visibly sensitive to:
  - octant: definite shift from maximal mixing;
  - hierarchy: relevant for the bound on $\theta_{13}$;
  - CP phase: impact on $\theta_{13}$ bound;
- with the present accuracy of atmospheric and long-baseline data, the sensitivity to subleading effects in the ATM sector ($\Delta m_{31}^2, \theta_{23}$) is completely dominated by atmospheric data.
Neutrino oscillations: where we are

- Updated global 6-parameter fit (including $\delta_{\text{CP}}$):
  - Solar: Cl + Ga + SK-I + SNO-leta (I+II) + SNO-III + BX-low + BX-high;
  - Atmospheric: SK-I + SK-II;
  - Reactor: Chooz + KamLAND;
  - Accelerator: K2K + Minos-DIS + Minos-APP;

- BPS09(GS): best-fit and 1$\sigma$ (3$\sigma$):
  \[
  \theta_{12} = 34.4 \pm 1.0 \left(\frac{3.2}{2.9}\right), \quad \Delta m^2_{21} = 7.59 \pm 0.20 \left(\frac{0.59}{-0.61}\right) \times 10^{-5} \text{ eV}^2, \\
  \theta_{23} = 42.3 \pm 5.3 \left(\frac{11.4}{-7.1}\right), \quad \Delta m^2_{31} = \begin{cases} 
  -2.40 \pm 0.11 \left(\frac{0.37}{-0.39}\right) \times 10^{-3} \text{ eV}^2, \\
  +2.51 \pm 0.12 \left(\frac{0.39}{-0.36}\right) \times 10^{-3} \text{ eV}^2,
  \end{cases} \\
  \theta_{13} = 6.8 \pm 2.6 \left(\frac{+6.4}{-3.6}\right), \quad \delta_{\text{CP}} \in [0, 360];
  \]

- BPS09(AGSS): same as above except:
  \[
  \theta_{12} = 34.5 \pm 1.0 \left(\frac{3.2}{2.8}\right), \quad \theta_{13} = 6.3 \pm 2.7 \left(\frac{+6.6}{-3.7}\right);
  \]

- $\theta_{13} \neq 0$:
  \[
  \begin{array}{c|cc}
  & \Delta \chi^2 & \text{Sol+Kam} & \text{Global} \\
  \hline
  \text{BPS09(GS)} & 1.52 & 2.02 \\
  \text{BPS09(AGSS)} & 1.06 & 1.73
  \end{array}
  \]

[Gonzalez-Garcia, MM & Salvado, in preparation]
II. Models with extra sterile neutrinos

The LSND problem

- LSND observed $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam ($E_\nu \sim 30$ MeV, $L \approx 35$ m);
- Karmen did not confirm the claim, but couldn’t fully exclude it either;
- the signal is compatible with $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations provided that $\Delta m^2 \gtrsim 0.1$ eV$^2$;
- on the other hand, other data give (at $3\sigma$):
  \[ \Delta m^2_{\text{sol}} \approx 7.6 \pm 0.6 \times 10^{-5} \text{ eV}^2, \]
  \[ |\Delta m^2_{\text{atm}}| \approx 2.5 \pm 0.4 \times 10^{-3} \text{ eV}^2; \]
- in order to explain LSND with mass-induced neutrino oscillations one needs at least one more neutrino mass eigenstate;
- WARNING: having enough $\Delta m^2$ is not enough. To make sure that the model works, one has to check explicitly that all the experiments can be fitted simultaneously.

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II. Models with extra sterile neutrinos

Four neutrino mass models

- Approximation: $\Delta m^2_{\text{SOL}} \ll \Delta m^2_{\text{ATM}} \ll \Delta m^2_{\text{LSND}} \Rightarrow 6$ different mass schemes:

- Total: $3\, \Delta m^2$, 6 angles, 3 phases. Different set of experimental data partially decouple:
II. Models with extra sterile neutrinos

(2+2): ruled out by solar and atmospheric data

- in (2+2) models, the fractions of $\nu_s$ in solar ($\eta_s$) and atmospheric ($1 - d_s$) oscillations add to one $\Rightarrow \eta_s = d_s$;

- $3\sigma$ allowed regions $\eta_s \leq 0.31$ (solar) and $d_s \geq 0.63$ (atmospheric) do not overlap; superposition occurs only above $4.5\sigma$ ($\chi^2_{PC} = 19.9$);

- the $\chi^2$ increase from the combination of solar and atmospheric data is $\chi^2_{PG} = 28.6$ (1 dof), corresponding to a PG $= 9 \times 10^{-8}$. [MM & Schwetz, PRD 68 (2003) 033020, hep-ph/0304176]
(3+1): tension between LSND and short-baseline data

- In (3+1) schemes the SBL appearance probability is effectively 2ν oscillations:
  \[ P_{\mu e} = \sin^2 2\theta \sin^2 \frac{\Delta m^2_{41} L}{4E}, \quad \sin^2 2\theta = 4 |U_{e4}|^2 |U_{\mu 4}|^2; \]

- disappearance experiments bound $|U_{e4}|^2$ and $|U_{\mu 4}|^2$;

- LSND is in conflict:
  - with other appearance experiments (Karmen & Nomad);
  - with all disappearance experiments.

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ESSS 2009, LUND, SWEDEN, 3/12/2009
The MiniBooNE experiment

- $E_\nu$ and $L$ very different from LSND (but similar $L/E_\nu$) \(\Rightarrow\) can check the oscillation solution of the LSND problem, not the signal itself;
- very peculiar results:
  - strong low-energy excess in $\nu_e$ (not $\bar{\nu}_e$);
  - weak mid-energy excess in $\bar{\nu}_e$ (could be 0).

\[\text{MB-}\bar{\nu}_e\]

\[\text{NuMI-}\nu_e\]
II. Models with extra sterile neutrinos

**Status of (3+1) models after MiniBooNE**

- (3+1) four-neutrino schemes fail because:
  - can’t reconcile *appearance* and *disappearance* data;
  - can’t explain the different $\nu_e$ (MB) and $\bar{\nu}_e$ (LSND) results;
  - can’t account for the low-energy $\nu_e$ event excess in MB.

$\Rightarrow$ (3+1) models are ruled out as explanation of SBL data.

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**Figure:**

- **BNB-MB($\nu$)**
- **BNB-MB($\bar{\nu}$)**
- **LSND**

**Plots:**

- $\Delta m^2_{41}$ vs $\sin^2(2\theta_{\mu\bar{e}})$ with 90% and 99% CL contours.

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Explaining the MiniBooNE excess with two sterile neutrinos

- With one extra sterile neutrino, $m_4$:
  \[ P_{\mu e}^{4\nu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2 \phi_{41} \text{ with } \phi_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}; \]
  - for large energy $P_{\mu e}^{4\nu}$ drops as $1/E^2$;
  - however, the low-energy MB excess is much sharper ($\sim 1/E^4$);
  \[ \Rightarrow \text{it is not possible to account for the MB excess with only one extra sterile neutrino.} \]

- On the other hand, with two extra neutrinos, $m_4$ and $m_5$:
  \[ P_{\mu e}^{5\nu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2 \phi_{51} + 8|U_{e4}U_{e5}U_{\mu 4}U_{\mu 5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} - \delta); \]
  - terms of order $1/E^2$ cancel if $\delta = \pi$ and $|U_{e4} U_{\mu 4}| \Delta m^2_{41} = |U_{e5} U_{\mu 5}| \Delta m^2_{51}$;
  \[ \Rightarrow \text{with two extra sterile states it is possible to fit the MB low-energy excess.} \]
II. Models with extra sterile neutrinos

Reconciling MiniBooNE and LSND in (3+2) models

- **Trick:** use the CP phase $\delta = \arg(U_{e4}^* U_{\mu4} U_{e5} U_{\mu5}^*)$ to differentiate $\nu$ (MB) from $\bar{\nu}$ (LSND):

  $$P_{\mu e}^{5\nu} = 4|U_{e4}|^2|U_{\mu4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2|U_{\mu5}|^2 \sin^2 \phi_{51} + 8|U_{e4} U_{e5} U_{\mu4} U_{\mu5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} - \delta)$$;

- note that $\delta = \pi + \epsilon$ and $|U_{e4} U_{\mu4}| \Delta m_{41}^2 \approx |U_{e5} U_{\mu5}| \Delta m_{51}^2$ to suppress MB probability.

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ESSS 2009, LUND, SWEDEN, 3/12/2009
Fitting all appearance data in (3+2) models

| data set       | $|U_{e4}U_{\mu4}|$ | $\Delta m_{41}^2$ | $|U_{e5}U_{\mu5}|$ | $\Delta m_{51}^2$ | $\delta$ | $\chi^2_{\text{min}}$/dof | gof |
|----------------|-------------------|-------------------|-------------------|-------------------|---------|--------------------------|-----|
| appearance (CPC) | 0.12              | 0.18              | 0.006             | 2.31              | –       | 95.8/86                  | 22% |
| appearance (CPV) | 0.080             | 0.39              | 0.029             | 1.10              | 1.1$\pi$| 82.5/85                  | 56% |

II. Models with extra sterile neutrinos

The doom of disappearance data

- As for (3+1) models, disappearance data imply bounds on $|U_{ei}|^2$ and $|U_{\mu i}|^2$ ($i = 4, 5$);
- these bounds are in conflict with the large values of $|U_{ei}U_{\mu i}|$ required by appearance data;
- again, a tension between APP and DIS arises:
  
  $\chi^2_{PG} = 17.5$ (4 dof) $\Rightarrow$ PG = $1.5 \times 10^{-3}$ [no MB];
  
  $\chi^2_{PG} = 17.2$ (4 dof) $\Rightarrow$ PG = $1.8 \times 10^{-3}$ [MB475];
  
  $\chi^2_{PG} = 25.1$ (4 dof) $\Rightarrow$ PG = $4.8 \times 10^{-5}$ [MB300];

- alternatively, compare LSND and NEV as in (3+1):
  
  $\chi^2_{PG} = 19.6$ (5 dof) $\Rightarrow$ PG = $1.5 \times 10^{-3}$ [before MB];
  
  $\chi^2_{PG} = 21.2$ (5 dof) $\Rightarrow$ PG = $7.4 \times 10^{-4}$ [after MB].

$\Rightarrow$ Conclusion: (3+2) models fail exactly as (3+1) did!

[MM & Schwetz, arXiv:0704.0107]
Conclusions

• **(3+0)** three-neutrino oscillations are now well established and accepted as an extension of the Standard Model. They can account for all the experimental results (except LSND and MiniBooNE) in a minimal way.

• **(3+1)** four-neutrino schemes are ruled out as explanation of LSND & MB because:
  − MB $\nu_e$ appearance data are incompatible with LSND $\bar{\nu}_e$ data;
  − the tension between LSND and NEV SBL data becomes more severe due to MB;
  − it is not possible to account for the low energy $\nu_e$ event excess in MB.

• **(3+2)** five-neutrino schemes
  + provide a good fit to LSND and to all the MB data samples through CP violation;
  + can account for the low energy $\nu_e$ event excess in MB;
  − fail to resolve the tension between appearance and disappearance data.

• **(3+3)** six-neutrino schemes do not offer qualitatively new effects. In particular, the global $\chi^2$ improves only marginally with respect to (3+2), hence the conflict between appearance and disappearance data remains.